Optimization of wind farm design taking into account uncertainty in input parameters

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Abstract

Optimization of wind farm design with risk assessment is presented in this paper. The net present value (NPV) is used to evaluate the yield of the laid-down capital of the wind farm. Monte Carlo simulation method is applied to obtain probability distribution of the objective function. The uncertainties of the wind speed and direction and power curve of the wind turbine are studied by incorporating them in the annual energy production (AEP) uncertainty, which can be directly translated into NPV uncertainty. Differential Evolution (DE) is used as the optimization algorithm.

Keywords: wind farm, siting, Differential Evolution, Monte-Carlo simulation

1 INTRODUCTION

The wind energy sector has developed rapidly in recent decades and experience has demonstrated the beneficial economic, environmental and social outcomes from the commissioning of wind power [1]. However, electricity produced by wind installations currently is more expensive compared to conventional power generation such as from fossil fuels or hydro power [2]. The incentive to increase attractiveness of wind projects to investors has resulted in the wind energy systems optimization being an active area of research [3]. The optimization of the wind farm design is one of the main topics for the reason that high improvements in cost efficiency can be achieved at the design stage of the wind farm project [4].

In the pioneering work of [5] on wind farm design optimization, the authors proposed a simple objective function to maximize energy output with minimum installation cost. The study demonstrated that the application of a genetic algorithm (GA) gives good quality results in maximizing the objective function value. Subsequently, a number of further articles have been published in the field of wind farm optimization. It should be noted that GA has been used most frequently [5-16], although other optimization techniques have also been employed. For instance, greedy heuristic [17, 18], simulated annealing [19], swarm optimization [20] and pattern search [21] have all been applied to achieve refined results.

Wind farm projects typically involve significant financial risks. The uncertainty in the energy production due to the intermittent nature of the wind and future costs and prices are the main contributors to the investment risk [2]. To date, few articles have considered risks when optimizing wind farms [23], [24]. Uncertainties arising from the wind and power curve models have been taken into account by the former for optimizing the capacity of the farm by looking at different wind scenarios. The latter investigates exclusively the influence of wind parameters uncertainty on the optimum layout on the basis of the Utility Theory.

The objective of this paper is to present an approach of finding the optimum locations, hub heights and rotor diameters of the wind turbines at a given site while taking into account risks caused by the uncertainties of the wind farm performance. The stochastic objective function uses statistical models of uncertainties of wind velocity and direction and the power output of individual turbines at given conditions. Objective function values are obtained by Monte Carlo simulation. Economical risks due to uncertainties of costs and future energy prices were ruled outside the scope of this paper; particularly the uncertainty of electricity price is largely dependent on energy policy decisions and thus very difficult to estimate. This paper therefore assumes that wind energy producer will receive price guaranteed with feed-in tariff.

The optimization problem is solved by differential evolution (DE), a type of evolutionary algorithm (EA). The optimized design should meet the interest of investors by having a high profit under the condition of a reasonable risk. Rather than using the mean value or the one with the highest probability, the objective of the algorithm is to maximize the profit for a chosen certainty level. It means we are looking at a boundary value, which will be exceeded with a given likelihood, reducing the risk to fail the expectations. A Matlab tool has been developed that optimizes the wind farm design according to the methodology described below.

2 Modelling of the Wind Farm

Wind farm model and assumptions made in the study are presented in this section.

2.1 Wind model

Wind behavior is intermittent, i.e. the wind speed and direction vary with time and height. For most sites, Weibull distribution is an efficient statistical representation of the wind conditions at a site [25].

$$f(V) = \frac{k}{c}\left(\frac{V}{c}\right)^{k-1} e^{-\left(\frac{V}{c}\right)^{k}}, \tag{1}$$

where V is the wind speed, m/s; and k, c are the shape and scale (m/s) parameters of the function, respectively. Both parameters of the Weibull function are dependent on the value of the mean wind speed $\bar{V}$ and standard deviation σ_V, which shows how dispersed the data (wind speed) is around the mean value [25].

$$k = \left(\frac{\sigma_V}{\bar{V}}\right)^{-1.086}, \tag{2}$$

$$c = \frac{\bar{V}}{\Gamma(1 + 1/k)}, \tag{3}$$

where Γ is a gamma function.

The wind speed increases with height and is influenced by the roughness class of the terrain and atmospheric stability. The following simple non-physical expression, called the power law, is widely used for estimating wind speed V at a certain height h. It gives best results when the height is above 50 m [26].

$$\frac{V}{V_0} = \left(\frac{h}{h_0}\right)^{\gamma}, \tag{4}$$

where γ is the wind gradient exponent (or exponent for wind shear) that shows the roughness of the terrain, V_0 is measured wind speed at a height h_0.

This empirical formula gives a reasonable result as long as the wind shear coefficient γ is selected correctly. Values of this parameter can change depending on complexity of the site, the wind direction, and time of the year [26].

2.2 Wake model

Compared with single turbine sites, wind farms give the advantages of reduced running and installation costs per turbine, but energy losses per turbine increase. The main reason for the lower efficiency of power production is the wake effect. The rotating blades of a turbine work as a barrier for the air stream, and consequently, the air flow behind the turbine is characterized by lower velocity and higher turbulence intensity [27]. This is a significant issue for wind farms as it decreases power production and increases mechanical loads on turbines placed in turbulent regions. It is important to estimate the expansion region of the wake and to calculate the deficit of the wind speed depending on the distance from the affecting turbine, as it directly influences the wind farm layout and choice of wind turbine type. Large spacing between turbines reduces the wake effect but increases the area needed for installation of the farm and thus other expenditure, such as land rents and infrastructure costs.

Simple empirical wake model developed by Jensen is used in the paper. The advantages of the model are its simplicity, low computation cost, and relatively effective estimations of wind speed losses behind the turbine [28].

The Jensen wake model [28], which assumes linearity of wake expansion, is presented in the following equations:

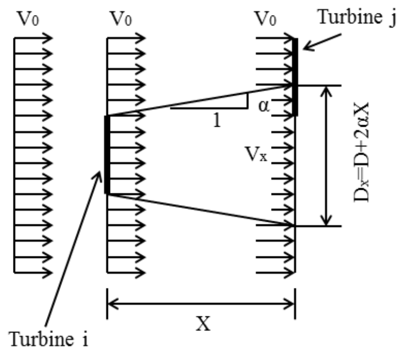

Figure 1: Schematic view of wake effect

$$V_X = V_0\left(1 - \frac{\left(1 - \sqrt{1 - C_T}\right)}{\left(1 + 2\alpha\frac{X}{D_0}\right)^2}\right), \tag{5}$$

$$D_x = D_0\left(1 + 2\alpha\frac{X}{D_0}\right), \tag{6}$$

where V_0 is the free wind speed before the turbine, m/s; C_T is the turbine thrust coefficient; α is the wake decay constant; X is the distance at which the wind speed is evalueted, m; and D_0 is the diameter of the blades, m.

The wake decay constant shows how the "shadow cone" increases in meters per meter behind the rotor, i.e., the turbulent area created by the turbine broadens. The typical value for wind farms on land is 0.075 m [25], but in the case of a wind park where several rows of turbines are installed, the opening angle of the wake cone increases after a turbine which is already within the wake of another turbine.

The thrust coefficient C_T is the relation of the thrust force (in other words the force with which the wind acts on the turbine) and dynamic pressure of the air flow multiplied by the swept area. A theoretical value of C_T = *8/9* is used in the paper [25].

The impact of several turbines on the energy output of a turbine downstream needs to be taken into account. The following equation is used:

$$V_i = \sqrt{V_0^2 - \sum_{j=1}^{N_i}\left(V_j^2 - V_{j,i}^2\right)}, \tag{7}$$

where N_i is the number of wind turbines influencing the turbine *i*; V_0 is the undisturbed wind speed, m/s; V_j is the wind speed at the turbine inlet, m/s; $V_{j,i}$ is the downstream wind speed of the *j* turbine at the position of the *i* turbine, m/s.

Estimation of the expansion region of the wake and calculating the deficit of the wind speed depending on the distance from the affecting turbine is important, as it directly influences the wind farm layout and choice of wind turbine type.

2.3 Terrain model

Wind resources depend on the complexity of the terrain, i.e. wind speed distribution changes with location and direction. Terrain elevation is not considered in this study meaning that wind is assumed to be homogeneous throughout the wind farm project land.

The terrain has a rectangular shape, where it is possible to add forbidden zones for wind turbine installation. The wind gradient exponent is chosen for flat open terrains equaling 0.15 [29]. The example terrain used in this study is depicted below.

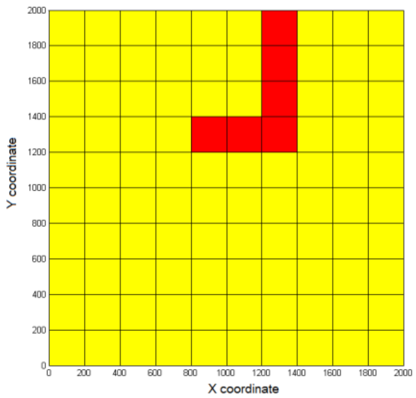

Figure 2: Terrain model

Allowed and forbidden zones for wind turbine installation are represented with yellow and red color, respectively.

2.4 Power curve model

The power curve is a characteristic of the turbine power output and depends on the type and size of the turbine, wind conditions at a site, and the period of time in operation. Electrical power produced by the wind turbine at a certain wind speed can be found by the following equation [25]:

$$P(V) = P_{kin}C_p\eta = \frac{1}{2}\rho AV^3C_p\eta, \tag{8}$$

where P_{kin} is the power of the flow; C_p is the power coefficient; η is the drive train efficiency.

However for this paper it is assumed that the power curve is provided by the manufacturer. Example of the power curve is depicted in Figure 6.

2.5 Economic model of the farm

Cash flows, time and risk are key factors determining the financial aspects of wind farm projects and a variety of techniques are used to evaluate the feasibility of projects. Previous research has shown that results of the wind farm optimization are highly sensitive to the choice of the objective function [11, 22]. Several studies suggest that net present value (NPV) is the most appropriate indicator for wind farm investment decision [14, 16], which is also one of the most universally accepted criteria.

The main components of the wind farms' economics such as initial investments $C_{inv,t}$, operation and maintenance (O&M) costs $C_{OM,t}$ and income I_t are considered for calculating NPV.

$$\mathrm{NPV} = \int_0^T \left(I_t - C_{inv,t} - C_{OM,t}\right) dt, \tag{10}$$

The income depends on the power produced by the turbines of the wind farm per year and cost of power at the electricity market. To analyze the cost-effectiveness of a project, future earnings should be conveyed to the present for comparison with investment costs.

$$I = \sum_{i,t=0}^{N,T} c_t \cdot \kappa_t \cdot AEP_{it}, \tag{11}$$

where N is the number of turbines; T is the project life time; c_t is the cost of energy for year t, €/MWh; AEP_{it} is the energy produced by i^{th} turbine at time t, MW; κ_t is the discounting parameter.

$$\kappa_t = \frac{1}{\alpha^t} = \frac{1}{\left(1+\frac{p}{100}\right)^t}, \tag{12}$$

p denotes the interest (discount) rate during the year t.

One of the assumptions made in the paper is that future electricity selling price is secured with feed-in-tariff over the life time of the wind farm project. Consequently, minimization of cost of energy and maximization of NPV should give the same optimum results. At the paper optimization of the NPV is performed, because it gives a clear picture to investor about the lay-down capital of investments.

Wind farm annual energy production (AEP) can be determined by the wind turbine power curve and the wind velocity frequency distribution at the hub height of the turbines [25]. For calculating the AEP, the average values of wind frequency distribution $f_{k,q}(V)$ and power output $P_q(V)$ are multiplied at 1 m/s intervals q of wind speed in each direction D. The product is summed over a range of wind speeds q starting from the minimum V_{in} and reaching to maximum V_{out} operational wind speed, and then summed for all directions D. The following equation for the calculation of the AEP considers the total effectiveness of the power output of the wind farm $w_{k,i}$ (considering wake losses) and the share of time in which the N turbine is in operation $T_{op,t}$.

$$AEP_t = T_{op,t}\sum_{d,i=1}^{D,N}\left(p_d w_{k,i}\sum_{q=V_{in}}^{V_{out}} P_q(V)\cdot f_{k,q}(V)\right), \tag{13}$$

The second component of the NPV, the capital cost of a project, includes the cost for design work, purchase of equipment, rent of land for onshore projects, civil works, electrical infrastructure costs, etc. Typical relations between expenditures for different capital cost components are given in [30]. To obtain the investment costs of turbines with different diameters and heights, data from [31] has been used.

And the last component is the costs for operation and maintenance (O&M). Like any other equipment, a wind turbine requires attendance during its lifetime. Costs for operation and maintenance (O&M) depend on the types and size of the turbines. Estimation of the O&M costs is very difficult, given that at this planning stage of the project many parameters are quite rough. O&M costs assumed to be fixed over the life time of the project [30].

3 Methodology

Based on the optimum solution, the design parameters of the wind farm are defined in the given area: i) number of wind turbines planned to be installed; ii) location of each turbine; iii) type and size of the wind turbines (rated power capacity, rotor diameter, hub height, etc.).

3.1 Existing issues

The problem of optimization of wind farm layout is not trivial for the following reasons:

- The problem is not separable: the optimum of each decision variable cannot be determined in isolation, but depends on the values of other variables.
- The objective function is likely to be highly multimodal.
- The decision variables include both discrete and continuous variables, which makes the problem also non-differentiable and prevents the use of traditional gradient-based optimization methods. The independent variables are uncertain.
- Depending on the number of variables, the number of possible wind farm design variants can reach a huge figure. An equation for calculating the total number of alternatives for n installed turbines is given below:

$$K_n = \frac{l!}{(l-n)!\,n!} h^n t^n, \qquad (14)$$

where *l* is the number of possible locations; *n* is the number of turbines planned to be installed; *h* is the number of hub height alternatives; *t* is the number of the turbine types. Growth of the number of variables entails significant increase of *K*.

3.2 Monte-Carlo simulation

Monte-Carlo simulation is applied to include risk in the economic model of the wind farm. This technique uses probability distributions of input parameters instead of assumed expected values of these parameters [32]. Calculations are repeated with samples of input parameters, which are provided with given distributions. The procedure allows the probability distribution for the desired output parameter to be obtained.

Implementation of the project is associated with a number of risks, in particular the uncertainty in energy production and future costs and prices. The scope of this work is to study the effect of the amount of energy produced at a profit, by incorporating uncertainty in the power curve and wind speed and direction distribution. All uncertainties incorporated in the model are assumed to be independent.

The wind statistical model [33] is used in this work, where the scale factor *c* is replaced by the mean wind speed $\bar{V}$:

$$f(V) = \frac{k\Gamma(1+1/k)}{\bar{V}} \left(\frac{V\Gamma(1+1/k)}{\bar{V}}\right)^{k-1} e^{-\left(\frac{V\Gamma(1+1/k)}{\bar{V}}\right)^{k}}, (15)$$

This model is used because the distribution of the mean wind speed is available while that of the scale factor is not.

The data [34] of mean wind speed demonstrate lognormal distribution (Figure 4), which is applied for the modeling.

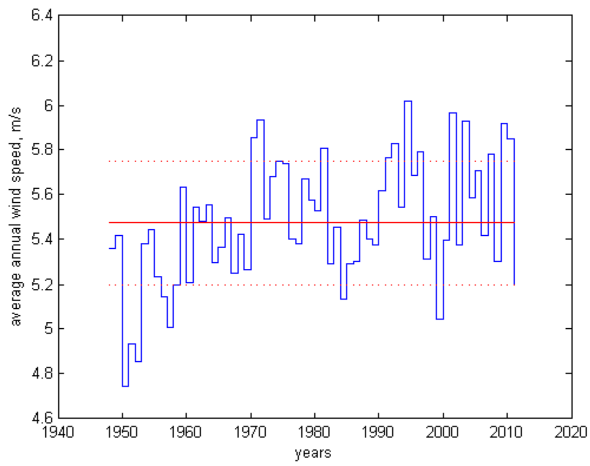

Figure 3: Annual deviation of the wind speed [34]

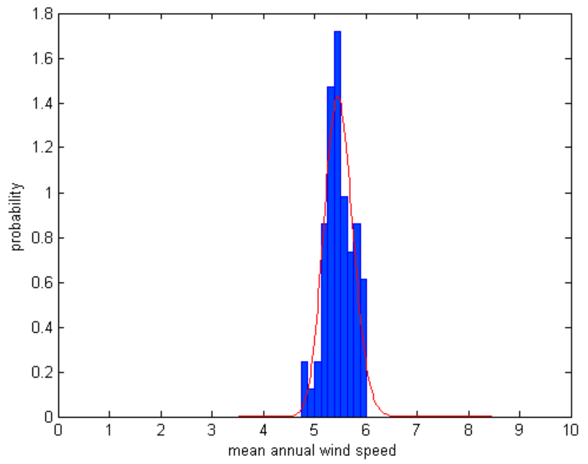

Figure 4: Annual mean wind speed distribution [34]

Two types of uncertainties can be distinguished. First of all, there are uncertainties inherent to physics, for example, due to the volatile nature of the wind. Model and measurement uncertainties form a second group. For instance, the estimation of the wind speed at the hub height contains the errors of measurement along with errors due to an imprecise formula for calculation. Likewise, the power curve provided by manufacturer is correct for conditions during the test only. The wind gradient, air density, turbulence intensity and control strategy cause the power output to differ from the measured values supplied by manufacturer [35, 36, 37].

Study [35] explicitly explains and defines the nature of uncertainties for the wind speed. The uncertainties from the wind are divided at four main categories: i) measurement, ii) long–term resource estimation, iii) wind resource variability and iv) site assessment uncertainties. By following recommendation of this study the overall uncertainty of the mean wind speed and Weibull shape parameter *k* has been obtained for flat terrain.

The work [33] demonstrates that the power curve uncertainty varies with wind speed as shown in Figure 5.

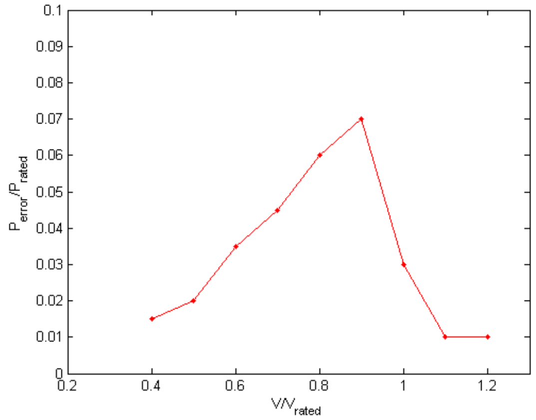

Figure 5: Normalized uncertainty in the power curve to normalized wind speed at hub height [30]

Uncertainty of the power curve production is assumed to have a normal distribution (Figure 6) [33].

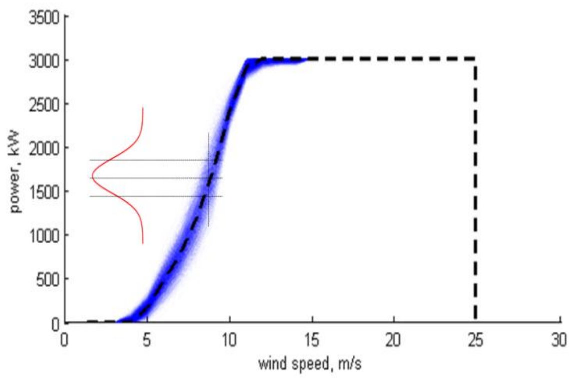

Figure 6: Power curve

Wind rose is an angular diagram which describes the frequency of the wind in geographical azimuth coverage. Simplified wind rose is depicted at the Figure 7. This model is used in this study to incorporate changes of the wind direction at a site. The wind rose is divided in sectors, which contain information of the probability of the wind occurrence in each sector (direction). Within each sector *d* the direction of the wind φ_d can change at an angle $\varphi_i \in (-45°, 45°)$ with a uniform probability distribution. Described stochastic model of the wind direction is used for the purpose of demonstrating the principle. If actual wind rose variation data for a site is available, it can be substituted for the crude model presented here, or the variability can be simply omitted from the model.

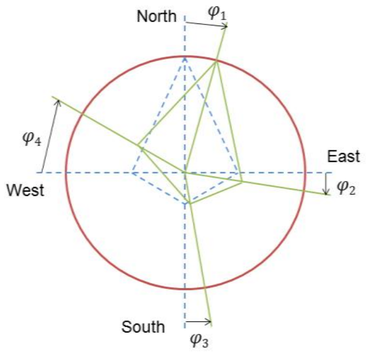

Figure 7: Simplified model of the wind rose, changing of the direction.

The AEP function is modified to stochastic as shown below:

$$AEP_t = T_{op,t} \sum_{d,i=1}^{D,N} \big[(p_d + \varepsilon_d) w_{k,i} \cdot \sum_{q=V_{in}}^{V_{out}} \big[P_q(V) + \varepsilon_{P,V}(V)\big] \cdot f_{k,q}(V)\big], \quad (16)$$

where

$$f_{k,q}(\mathrm{V}) = \frac{(k+\varepsilon_k)\Gamma(1+1/(k+\varepsilon_k))}{\bar{V}+\varepsilon_{\bar{V}}} \cdot \left(\frac{(\bar{V}+\varepsilon_{\bar{V}})\Gamma(1+1/(k+\varepsilon_k))}{\bar{V}+\varepsilon_{\bar{V}}}\right)^{(k+\varepsilon_k)-1} \cdot \mathrm{e}^{-\left(\frac{(\bar{V}+\varepsilon_{\bar{V}})\Gamma(1+1/(k+\varepsilon_k))}{\bar{V}+\varepsilon_{\bar{V}}}\right)^{(k+\varepsilon_k)}}, \quad (17)$$

$$\varepsilon_{\bar{V}} \sim lnN\left(0, \sigma_{\bar{V}}^2\right), \quad (18)$$

$$\varepsilon_k \sim N(0, \sigma_k^2), \quad (19)$$

$$\varepsilon_d \sim U(a, b), \quad (20)$$

$$\varepsilon_{P,V} \sim N\left(0, \sigma_{P,V}^2\right), \quad (21)$$

To handle the stochastic nature of the objective function, the number of samples should be large enough to give the same optimized value after each run or to give values with a certain confidence.

3.3 Objective function

The mathematical formulation of the optimization problem is as follows, with explanations regarding it given below:

Max $\quad NPV(x_i, y_i, type_i), \quad (22)$

S. t. $\quad x_{min} < x_i < x_{max}, \quad (23)$

$y_{min} < y_i < y_{max}, \quad (24)$

$type_i \in \mathbb{N}: type_i = [1, n], \quad (25)$

$distance\ between\ turbines > L_{min} = 2(r_i + r_j), \quad (26)$

$(x_i; y_i) \notin fobidden\ zone, \quad (27)$

where x_i, y_i, $type_i$ are the parameters (coordinates and type) of each *i* wind turbine to be optimized, r_i, r_j are the radii of the neighboring turbines *i, j* rotors, L_{min} is the minimal allowed distance between turbines, *forbidden zone* is an area where installation of the wind turbine is not possible.

Firstly, trial locations of the wind turbines should lie within borders of the grid eq (23) and eq (24).

Secondly, two types of constraint are inherent in the model. The first one eq (26) defines the minimum acceptable distance between turbines. Wind turbine located in the wake zone of another turbine experiences higher mechanical loads due to high TI intensity in the wake zone. This can lead to higher failure frequency in wind turbine components. In this study it is assumed that each turbine has a round shaped forbidden zone around it, which can be equal to two diameters of the turbine rotor. Second constraint eq (27) defines forbidden zones at the site, where turbines cannot be placed, for instance, zones close to buildings or roads.

3.4 Optimization algorithm

Differential evolution (DE), a type of evolutionary algorithm, was used in the optimization. Evolutionary algorithms (EAs) are stochastic population-based global optimization algorithms that provide robust convergence to a global optimum even if the objective function is multimodal, multiconstrained, non-continuous, non-differentiable or noisy. The cost of the reliability is a far greater computational time than what would usually be required with mathematical derivative-based optimization methods, or even single-point direct methods not based on derivatives.

Optimization by EAs is based on mimicking the natural selection process, dealing not with single points in the optimization space, but populations of trial solutions competing with each other for survival and chance of reproducing offspring (new trial solutions) to the population of next generation, and applying random mutations to ensure diversity of solutions and thorough searching through the objective function. The better the objective function value of a trial solution, the better its chance of passing the selection and surviving to the next generation. Genetic algorithms apply selection pressure also in the selection of parent solutions for recombination to produce new trial solutions.

Traditional DE is a real-valued optimizer where the search relies mainly on mutation and offspring selection without parent selection. Discrete variables are dealt by a coding scheme where allowable range of real values is divided to slots, and any value from one slot translates to a single discrete value to be used in the objective function evaluation.

An important characteristic of DE is that the magnitude of mutations is adjusted by basing it on a difference vector between two randomly chosen vectors r_1 and r_2. This ensures that as the population converges towards the optimum and the search proceeds from finding the right area to fine-tuning the solution, the mutations become gradually smaller.

In the original DE a new generation of vectors (candidate solutions) is produced from old generation by having each vector serve once as a target vector, and surviving to new generation if it wins the comparison with a trial vector *u* generated by crossover between a noise vector *v* and the target vector itself. Decision variable values of *u* are taken from *v* at probability defined by the tuning parameter CR, otherwise from the target vector. The noise vector *v* is created by differential mutation from a base vector x_0 chosen from generation *G* by adding to it the difference vector r_1-r_2 scaled with an adjustable parameter *F*. This form of DE is known as *DE/rand/1/bin* for *random* selection of base vector perturbed in differential mutation by *1* vector difference followed by *bin*omial crossover with target vector.

In the optimization of wind farm configuration considered in this study, each turbine is described with three variables, *x* and *y* coordinates defining the location, and a third variable defining the type, resulting in a total of $3n_{WT}$ decision variables where n_{WT} is the maximum number of turbines in the wind farm. The objective function is clearly not separable, i.e. the 'goodness' of one variable value cannot be determined in isolation but depends on other variables. For such problems the differential mutation provides a rotationally invariant search easily able to move in multiple variable axes simultaneously, which is problematic for crossover-reliant GA's. Rotational dependence is introduced by crossover even in DE unless CR=1, but arithmetic linear recombination provides means of using recombination while avoiding rotational dependence. A *DE/rand/1/either-or* algorithm [38] generates the trial vector *u* by either taking the noise vector *v* as such, or by a recombination from three randomly chosen individuals r_1, r_2 and r_3,

$$u = r_1 + K(r_2 + r_3 - 2\, r_1), \tag{28}$$

where *K* is a tuning parameter for the algorithm. In this implementation the crossover parameter CR is replaced by probability P_F of using differential mutation rather than arithmetic recombination. This variant has shown good combination of robustness, speed and ease of tuning in the optimization of highly multi-modal test functions [38], and was used in this study. In order to obtain faster convergence, each trial vector is evaluated with 5000 Monte Carlo samples. Mean function evaluation time took 10-12 second and accuracy of the estimation was within 1% with 5000 samples.

Initial population is generated randomly with uniform probability distribution within the allowed range for each decision variable. Invalid population members were not allowed for the initial population: in case of an invalid initial candidate, another random candidate is created repeatedly until a valid result is obtained.

During the run of the algorithm, an invalid candidate solution violating any constraints is never allowed to replace a valid one, and objective function values are thus not calculated for invalid candidates. If a candidate violates only boundary constraint(s) (eq (23), (24) and (27)) of the type $x_{i,min} < x_i < x_{i,max}$, the algorithm attempts to fix the candidate before rejecting it. This is performed by reflecting the illegal decision variable value(s) back to within boundaries, i.e. if value of variable x_i in a trial solution is $x_{i,max}+\Delta x_i$ or $x_{i,min}-\Delta x_i$ these would be replaced by $x_{i,max}-\Delta x_i$ or $x_{i,min}+\Delta x_i$ respectively. In the case when distance between turbines L_T is smaller than the minimum allowable distance $L_{min} = 2(r_i + r_j)$ by $\Delta L_T = L_{min}-L_T$, the algorithm attempts to fix the member by moving one turbine away from the other so that new distance will be $L_T = L_{min} + rand(1)\Delta L_T$.

4 Case study

The case with 10 turbines is considered. Terrain is assumed to be flat, as described in chapter 2.3. Turbine type could be chosen from four different 3 MW turbines differing in hub height and rotor diameter. The cost data for the four types were determined by using the cost distribution for different turbine components from [30] for a 5 MW turbine and an assumed 1200 €/kW installed cost for such turbine as a basis, and then applying the scaling formulas from [31] to obtain component costs for the different 3 MW turbines. The resulting turbine costs are listed in Table 1 below.

Table 1: Turbine costs

	Type 1	Type 2	Type 3	Type 4
Hub height H, m	100	100	120	120
Rotor diameter, m	50	60	50	60
Total turbine installation cost, €	4.5	5.8	4.8	6.2

Costs of O&M is assumed to be 10 € per MWh of energy produced over the life time of the wind farm [2].

It is assumed that electricity selling price is guaranteed with feed-in-tariff over life time of the project [39].

Table 2: Selling price for wind energy produced

Period of time	Price
$t \leq 3\ a$	105.3 €/MWh
$3\ a < t \leq 12\ a$	83.5 €/MWh
$12\ a < t \leq 20\ a$	60 €/MWh

Wind speed histogram and wind rose for the site are shown at the Figure 8 and Figure 7, respectively. Simplified wind rose is used, which is divided on 4 sectors. Following probabilities of occurrence wind speed have been used: North 50%, East 20%, South 10% and West 10%.

The project life time is assumed to be 20 years and operational time of the wind park is 90% of a year. An interest rate of 5% is used.

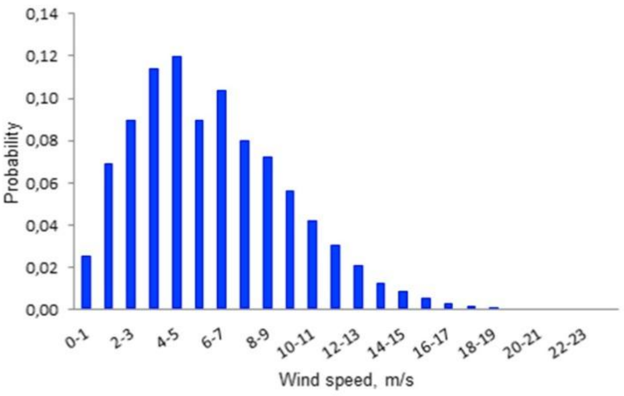

Figure 8: Wind histogram

5 Results

Convergence history of the 10 turbine case with population size NP=300 as presented in Figure 9 below. Solid line represent the change of best and dotted lines the change of mean objective function value in the population. Parameters of P_F=0.6 and F=0.7 are used. The tuning parameter K in eq (28) was estimated from K=0.5(F+1.0) on the basis of recommendation in [38].

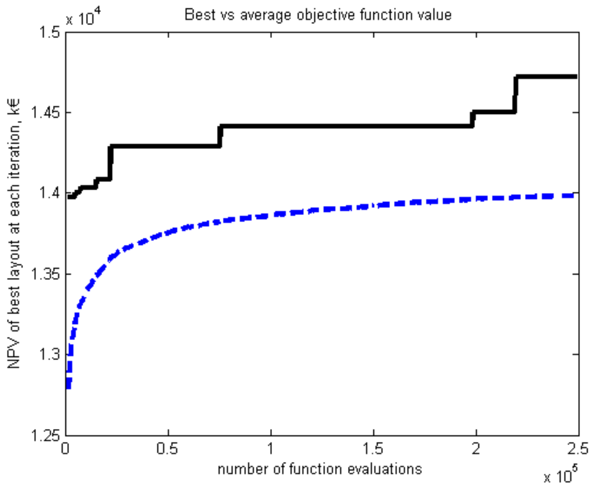

Figure 9: Convergence history

Solid line represent the change of best and dotted lines the change of mean objective function value in the population.

In the initial entirely random population the mean objective function value was 8.864 M€. From Figure 9 it can be seen that the population quickly and significantly improved from this value, but the rate of increase of the objective function value then rapidly slowed down. After $2.5{\cdot}10^5$ objective function evaluations, it is clear that the population is not yet fully converged to an optimal value. Further improvements that would be significant considering the uncertainties involved would appear unlikely, however.

The best solution obtained is demonstrated in Figure 10 below.

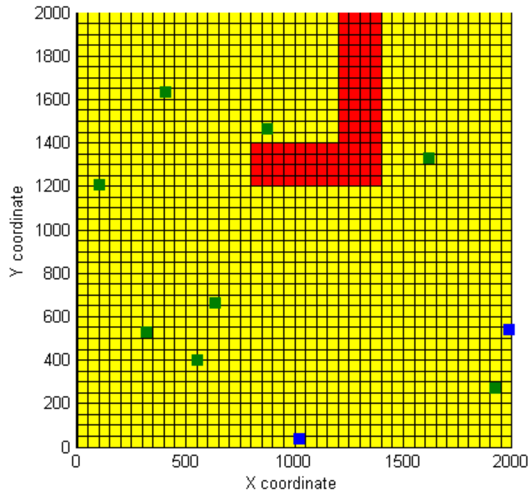

Figure 10: Optimized wind farm design after $2.5 \cdot 10^5$

With green and blue colors are shown the turbines of type 1 and type 2, respectively.

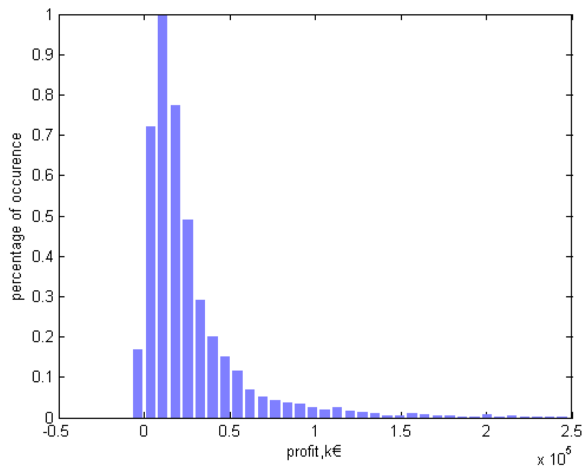

Figure 11: Probability distribution function of optimized wind farm

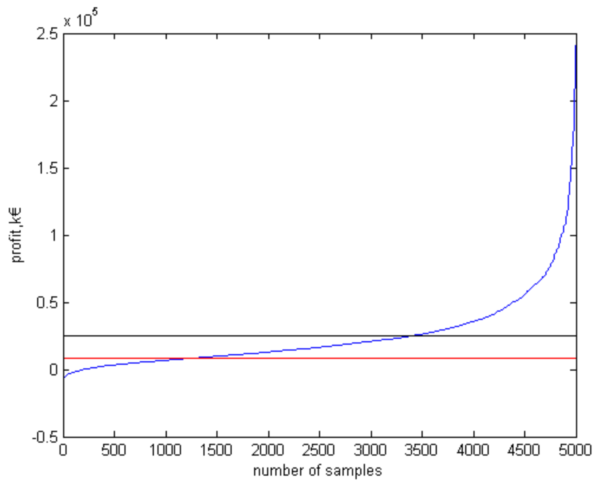

Figure 12: Cumulative provability density function of optimized wind farm

Figure 11 demonstrates probability distribution function (pdf) of an optimized wind farm. Throughout the optimization, the pdf of different wind farm layouts do not change significantly. It can be concluded that the distribution depends strongly on the wind distribution of the wind farm, which in this paper is assumed homogeneous. In Figure 12 is depicted the cumulative probability density function of optimized wind farm, where black and red line show mean and 75% of certainty level value of the objective function, respectively.

6 Conclusions

A method for considering uncertainties and minimizing risks in wind farm projects is presented in this study. Optimization is performed to maximize the minimum profit expected at 75% of certainty level, with Monte Carlo sampling used to obtain the performance of a candidate solution, and by differential evolution used as the optimization algorithm.

Due to the large number of Monte Carlo simulations required to obtain the objective function value of each candidate the optimization process is computationally considerably more demanding than what it would be if the objective was to maximize simply the expected profitability. This problem is further exacerbated by the large number of objective function evaluations required by DE to find the optimum. While the calculation time could be reduced by first estimating the objective function value with a smaller number of samples, some of this advantage is probably lost through the noise that this method adds.

The either-or DE strategy used here appears to be able to find an optimum solution, but the process is very slow and there are considerable difficulties in creating feasible individuals. Modifying the algorithm to be able to fix illegal candidate solutions proved to be vital for obtaining useful improvement rate, and even then full convergence to a single point could not be achieved in a reasonable amount of objective function evaluations.

In future work an attempt should be made to improve the convergence performance. The most obvious step is to investigate the sensitivity of the algorithm to the tuning parameters. Although DE and specifically the either-or strategy was presumed to be a suitable optimization method, other strategies of DE and other metaheuristics could provide better performance. A different coding scheme than using Cartesian coordinates of each turbine directly as decision variables could also yield improvements, as well as improved ways of fixing infeasible solutions.

In final conclusion, the DE implementation presented in this paper appears to be able to optimize the wind farm configuration, but only after a considerable amount of computation time. Clearly in future work effort should be made to improve the performance of the optimization algorithm.

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