Development and Testing of a Branch Current Based Distribution System State Estimator

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***Abstract*- The recent increase of distributed generation has forced many distribution network operators to develop distribution automation and active network management. Many active distribution network management functions need accurate real-time estimates of the network state. In this paper, a distribution network state estimation algorithm is developed and used in conjunction with coordinated voltage control. The state estimator utilizes equality constrained weighted least squares optimization and includes bad data detection. The state estimator is tested with MATLAB simulations, real-time digital simulator and in a real distribution network.**

***Index Terms* -- Bad data detection, distribution system state estimation, equality constraints, testing, weighted least squares.**

I. Introduction

The purpose of distribution system state estimation (DSSE) is to obtain the best possible estimate of the network state by processing available information. Nowadays DSSE relies mainly on substation measurements, network data and load profiles. The substation measurements include real-time measurements of busbar voltages and feeder current or power flows. With these measurements it is possible to adjust the feeder loads accurately, but the load distribution inside the feeders remains uncertain.

There is a need for more accurate DSSE because the amount of distribution automation and active control is constantly increasing. Active distribution network management functions such as voltage level management, control of distributed generation, reactive power regulation, feeder reconfiguration and restoration, and demand side management require accurate real-time estimates of network voltages and line flows. Especially the increase of distributed generation is an important driver for state estimation development [1].

DSSE can be made more accurate by adding measurements to the distribution network and using advanced state estimation methods. In the last 15 years, several new DSSE methods have been proposed in the literature [1]–[4]. Many of them are based on the weighted least squares method, but the selection of state variables varies. Some are using node voltages [1], [2] as state variables whereas others have chosen to use branch currents [3], [4].

In order to make DSSE more accurate, we developed a branch current based distribution system state estimator exploiting equality constrained weighted least squares optimization [5]. The state estimator was formulated to utilize all real-time current, power and voltage measurements available in a distribution network. The developed state estimator was written into a MATLAB program, and its performance and the effect of the additional current, power and voltage measurements were tested with MATLAB simulations.

In this paper, the state estimator is further developed by adding bad data detection using state estimation residuals. Furthermore, the state estimator is coupled with a coordinated voltage control algorithm [6] and tested in a Real-Time Digital Simulator (RTDS) and in a real distribution network.

This paper will first revise the formulation of the developed DSSE method and introduce the used bad data detection method. Thereafter, test results from MATLAB and RTDS simulation and real-life demonstration are presented. The test results are discussed and conclusions are given at the end.

II. Formulation

A. Main algorithm

The state estimation algorithm in this paper is based on the method presented by Wang and Schulz [4]. The algorithm uses the magnitudes and phase angles of branch currents as state variables. The benefit of using current magnitudes as state variables is that current magnitude measurements, which are the dominating measurement types in distribution systems, correspond directly with state variables. The algorithm uses weighted least squares (WLS) estimation to determine the most likely state of the network. In WLS estimation the goal is to minimize the weighted sum of squared measurement residuals. Measurement residual is the difference between measured and estimated value and each residual is weighted with the variance (accuracy) of the corresponding measurement.

Some modifications were done to the original algorithm. The algorithm was altered to use equivalent single phase circuits and equality constraints were added to handle the zero-injection measurements. The use of equality constraints helped to avoid the ill-condition problems arising from the combination of high and low weights associated to zero-injection and pseudo load measurements. The equality constrained WLS problem can be solved by using the method of Lagrange multipliers [7]. In the method of Lagrange multipliers the constrained minimization problem is solved by minimizing the Lagrangian function

$$L(\boldsymbol{x},\boldsymbol{\lambda}) = \frac{1}{2}[\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x})]^T \boldsymbol{R}^{-1}[\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x})] + \boldsymbol{\lambda}^T \boldsymbol{c}(\boldsymbol{x}), \quad (1)$$

where $\boldsymbol{x}$ is the state vector
$\boldsymbol{\lambda}$ is the Lagrange multiplier vector
$\boldsymbol{z}$ is the measurement vector
$\boldsymbol{h}(\boldsymbol{x})$ is the measurement function
$\boldsymbol{R}$ is the measurement covariance matrix ($\boldsymbol{R} = \mathrm{diag}[\sigma_1^2 \quad \sigma_2^2 \quad \cdots \quad \sigma_N^2]$ where σ_i^2 is the variance of the measurement i)
$\boldsymbol{c}(\boldsymbol{x})$ is the zero-injection measurement function.

The minimization problem can be solved by differentiating $L(\boldsymbol{x},\boldsymbol{\lambda})$ partially with respect to $\boldsymbol{x}$ and $\boldsymbol{\lambda}$ and setting the differentials to zero. This yields the following equations:

$$\frac{\partial L(\boldsymbol{x},\boldsymbol{\lambda})}{\partial \boldsymbol{x}} = -\boldsymbol{H}^T \boldsymbol{R}^{-1}[\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x})] + \boldsymbol{C}(\boldsymbol{x})\boldsymbol{\lambda} = 0 \quad (2)$$

$$\frac{\partial L(\boldsymbol{x},\boldsymbol{\lambda})}{\partial \boldsymbol{\lambda}} = \boldsymbol{c}(\boldsymbol{x}) = 0 \quad (3)$$

where $\boldsymbol{H} = \frac{\partial \boldsymbol{h}}{\partial \boldsymbol{x}}$ and $\boldsymbol{C} = \frac{\partial \boldsymbol{c}}{\partial \boldsymbol{x}}$ are the Jacobian matrices.

Equations (2) and (3) form a system of equations which can be solved iteratively by the Newton–Raphson method. At each iteration, the incremental change to the state vector ($\Delta\boldsymbol{x}$) is calculated with equation

$$\begin{bmatrix} \Delta\boldsymbol{x} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}^T\boldsymbol{R}^{-1}\boldsymbol{H} & \boldsymbol{C}(\boldsymbol{x})^T \\ \boldsymbol{C}(\boldsymbol{x}) & 0 \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{H}^T\boldsymbol{R}^{-1}[\boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x})] \\ -\boldsymbol{c}(\boldsymbol{x}) \end{bmatrix}. \quad (4)$$

Once the calculation has converged, the node voltages can be determined with a forward sweep calculation.

B. *Bad data detection*

Bad data detection is an essential part of any state estimator. State estimators must be able to detect, identify and remove bad data from the measurement set. Measurements may contain errors due to various reasons. Meters can have biases, drifts or wrong connections. Telecommunication system failures can also lead to large deviations in recorded measurements.

Some measurement errors are easy to detect with simple logical rules. For example, negative voltage and current magnitudes and measurements, which are several orders of magnitude larger or smaller than expected, are easily recognized as bad data. Unfortunately, not all types of bad data are detected that easily. However, in more indistinct cases, other detection methods can be utilized.

In WLS state estimation, the bad data detection can be made by examining the measurement residuals. This has to be done after the estimation process. The bad data detection is essentially based on the statistical properties of the residuals. One of the most used bad data detection methods is the *Largest Normalized Residual* r_{max}^N -test. This test is composed of the following steps [8]:

1. Solve the WLS estimation and obtain the elements of the measurement residual vector ($\boldsymbol{r}$):

$$\boldsymbol{r} = \boldsymbol{z} - \boldsymbol{h}(\boldsymbol{x}) \quad (5)$$

2. Compute the normalized residuals ($\boldsymbol{r}^N$):

$$\boldsymbol{r}^N = \frac{|\boldsymbol{r}|}{\sqrt{\boldsymbol{\Omega}_{\mathbf{ii}}}}, \quad (6)$$

where $\boldsymbol{\Omega}_{\mathbf{ii}}$ is $diag(\boldsymbol{\Omega})$
$\boldsymbol{\Omega}$ is $Cov(\boldsymbol{r})$.

3. Find the largest normalized residual ($\boldsymbol{r}_{max}^N$).
4. If $\boldsymbol{r}_{max}^N > c$, then the corresponding measurement is erroneous. Here, c is a chosen detection threshold, for instance 3.0.
5. If bad data is detected, eliminate the faulty measurement from the measurement set and go back to step 1.

The faulty measurements are eliminated one by one. After each elimination, WLS state estimation procedure is repeated.

The largest normalized residual test can detect bad data if the removal of the corresponding measurement does not render the system unobservable. It is possible to identify all cases of single bad data where the faulty measurements are not critical or belong to a critical pair or critical k-tuple. Critical measurements are those measurements whose removal would cause the system to become unobservable. A critical pair and k-tuple contain two or more measurements, respectively, whose simultaneous removal would make the system unobservable.

In the case of multiple bad data, only part of the measurements errors can be identified. Faulty measurements with weakly correlated measurement residuals can be identified. If the measurement residuals are strongly correlated, the bad data can be identified only in the case of non-conforming bad data. If the identification of faulty measurement fails, the largest normalized residual test can incorrectly remove a faultless measurement.

Because our state estimator is based on equality constrained WLS estimation, the measurement residual covariance matrix can not be solved as usual. Solution for this problem can be found from [7]. In equality constrained state estimation the measurement residual covariance matrix $\boldsymbol{\Omega}$ is equal to

$$Cov(\boldsymbol{r}) = \boldsymbol{R}^{-1} - \boldsymbol{H}\boldsymbol{E}_1\boldsymbol{H}^T, \quad (7)$$

where $\boldsymbol{E}_1$ is the upper left corner of the inverse of $\boldsymbol{F}$.

$$\boldsymbol{F}^{-1} = \begin{bmatrix} \boldsymbol{H}^T\boldsymbol{R}^{-1}\boldsymbol{H} & \boldsymbol{C}(\boldsymbol{x})^T \\ \boldsymbol{C}(\boldsymbol{x}) & 0 \end{bmatrix}^{-1} = \begin{bmatrix} \boldsymbol{E}_1 & \boldsymbol{E}_2{}^T \\ \boldsymbol{E}_2 & \boldsymbol{E}_3 \end{bmatrix}, \quad (8)$$

where $\boldsymbol{C(x)}$ is the Jacobian matrix of the equality constraint function.

The problem with measurement residual based bad data detection is that it requires a certain amount of redundancy from the measurement configuration. In distribution networks, the number of measurements and thus also the redundancy level is very limited. In this paper, load models are used as load pseudo-measurements. With these artificial measurements it is possible to detect and identify rough errors in real measurements.

III. Testing

A. *MATLAB simulations*

The above-presented algorithm was written into a MATLAB program, and its performance was tested with MATLAB simulations. IEEE 37-bus radial test feeder [9] was used in the simulations. The following modifications were made to the test feeder:

1) The voltage regulator was omitted.
2) All the loads were changed into constant PQ loads.
3) All the unsymmetrical loads were changed into symmetric three-phase loads and the feeder was modelled with an equivalent single phase circuit. This is a common simplification in Finnish distribution network calculation.
4) The nodes were renumbered for clarity.

The one-line diagram of the modified test feeder is shown in Fig 1.

The test feeder was assumed to have a basic set of measurements: active and reactive power flow measurements at the beginning of the feeder, a voltage measurement at the node 1 and pseudo-measurements at the load nodes. The measurement accuracies were set to ±1 % for the power flow measurements and ±0.2 % for the voltage measurement (with a 95 % confidence level). The pseudo-measurements were given a relative standard deviation of 50 % in the area 1 and 20 % in the area 2.

Simulations comparing the proposed and existing Finnish DSSE methods were conducted. In the existing DSSE methods [10] only feeder line flow measurements are used to correct the load estimates and the difference between the estimated and the measured feeder power flow is distributed to the load estimates in relation to their standard deviations. The existing DSSE method was also modelled into the MATLAB.

The simulations were performed by first varying the loads normally according to the pseudo-measurement standard deviations. Then the true state of the feeder was calculated using a load flow program. The power flow and voltage measurements were created from the true states by varying them normally according to the corresponding measurement accuracy. Finally, the state estimates were computed and the estimation errors were calculated for node voltages by comparing the estimates with the true values. This procedure was repeated 10000 times and average errors were calculated.

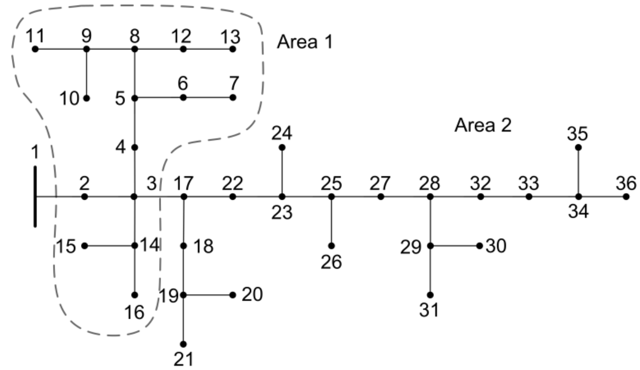

Fig. 1. One-line diagram of the modified test feeder.

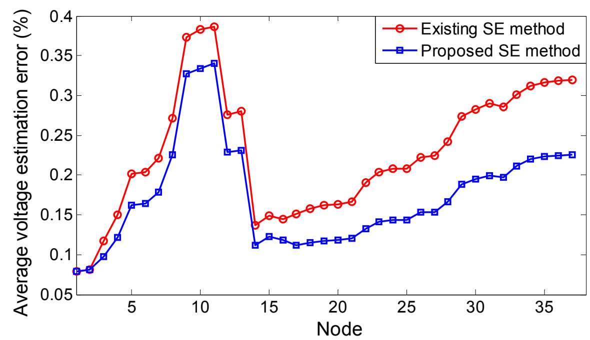

Fig. 2. Estimation accuracy comparison.

The proposed state estimation method provided 24 % smaller average voltage estimation error. The difference is shown in Fig. 2. Simulations were also done with additional power, current and voltage measurements to study their effects on the state estimation accuracy. Some of these results are published in [5].

B. *RTDS simulations*

In the next testing phase, the state estimation algorithm was coupled with a coordinated voltage control algorithm [6] and the resulting MATLAB prototype software was tested in RTDS environment. The purpose of RTDS simulations was to verify the correct operation of the prototype software before it is demonstrated in a real distribution network.

The coordinated voltage control algorithm aims to keep the network voltages between acceptable limits by controlling available active resources. In these simulations, it controls substation voltage and DG reactive power by changing the set points of substation automatic voltage control relay and DG automatic voltage regulator. The coordinated voltage control algorithm uses the results of the state estimation as inputs. In this paper, we concentrate on the state estimation part of the RTDS simulations. Simulation results from the coordinated voltage control point of view can be found in [11].

The simulation arrangement

In these simulations, RTDS is used to emulate a real distribution network. The simulation arrangement is depicted in Fig. 3. RTDS consists of hardware and software. The

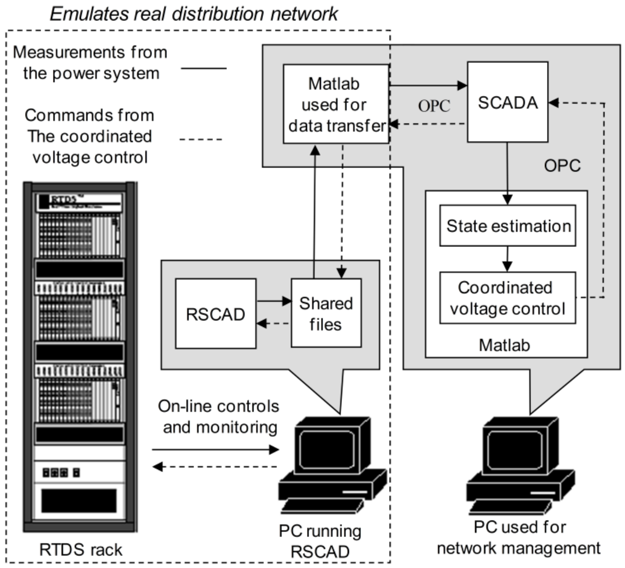

Fig. 3. RTDS simulation arrangement.

hardware is used to solve power system equations in real-time and is installed in a rack. The RSCAD software is run on an external computer and is used to construct the power system models and to control the simulations. The simulated network is controlled using commercial SCADA software (ABB MicroSCADA Pro SYS 600) and the prototype software containing both state estimation and coordinated voltage control algorithms. Measurement signals from the simulated network are transferred to SCADA. Data transfer between RSCAD and SCADA is realized using shared files.

The simulation network

The simulation network is constructed to correspond to the network in the forthcoming real-life demonstration. The network consists of two medium voltage feeders and contains one relatively large hydro power plant. The RTDS simulations are done with a three-phase network model. A reduced version of the real network model is used because of RTDS limitations. A single-phase representation of the simulation network is shown in Fig. 4.

The simulation network includes active and reactive power flow measurements at the beginning of each feeder and at the hydro power plant. Voltages are measured from the substation and from the power plant. The power plant breaker status is also monitored. Loads are modelled as symmetrical static constant power loads. In state estimation, the load pseudo-measurements are given a 10 % relative standard deviation. The distribution lines are modelled in both RSCAD and in the state estimator using a nominal π-model.

Simulation results

First, the state estimation results where compared to the monitored values in RSCAD to verify the accuracy of load flow calculation embedded in to the state estimator. When given ideal error-free measurements as inputs, the differences

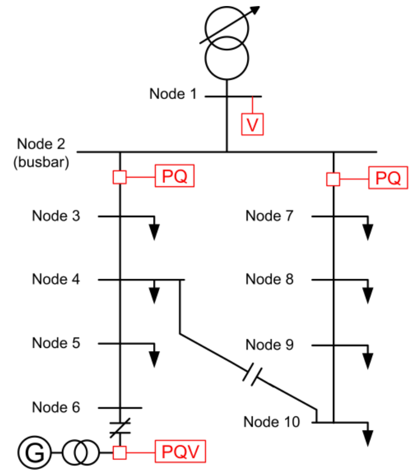

Fig. 4. RTDS simulation network.

in estimated and monitored node voltages were smaller than 0.01 %.

During the first set of RTDS simulation we noticed that the state estimation did not always converge when given highly conflicting inputs. Conflicting inputs can be caused, for example, by input synchronization errors. Synchronization errors were detected also in the RTDS simulations. Sometimes, when the power plant was disconnected from the network by opening the power plant breaker, the changed breaker status information reached the state estimator before the feeder power flow measurements had changed to correspond to the new topology. Bad data detection was added to the state estimator to tackle this problem.

The simulation network has a very low measurement redundancy, therefore detecting bad data is difficult. Only errors in power plant voltage measurement can be detected and identified directly from the measurement residuals. The feeder power flow measurements form critical pairs with the feeder load pseudo-measurements and with power plant power flow measurements. These groups of critical pairs are denoted here as *critical groups*. Removal of any measurement in a critical group would make the rest of the measurements critical. All the measurements in a critical group have equal normalized residuals, hence the erroneous measurement can not be identified. In order to identify faulty feeder power flow measurements we have to assume that the measurement errors can not be located in the load pseudo-measurements or power plant power flow measurements.

For example, if the load feeder reactive power flow measurement (Q_{27}) is erroneous, then that measurement and all reactive load pseudo-measurement on the same feeder (Q_7–Q_{10}) have identical normalized residuals. This is shown in Table 1. In order to identify the bad data, we have to assume that only the highlighted measurements in Table I can contain errors.

In RTDS simulations, the bad data detection threshold was set to 3.0, which is a typical bad data detection threshold in transmission system state estimation. With this threshold value, the measurement Q_{27} can vary between 2.32 and 3.39 p.u. without being suspected as bad data. In Table I, the measurement Q_{27} is outside this range, its normalized residual is larger than 3.0 and it is identified as bad data.

In the case of the previously mentioned input synchronization error, the bad data detection fails because the state estimation does not converge. This problem was solved by first detecting the existence of bad data from the non-convergence and then running the state estimation again without the feeder power flow measurements. After the new pseudo-measurement based state estimate was calculated, the normalized residuals were calculated for the feeder power flow measurements. Measurement with the largest normalized residual was identified as bad data and removed from the measurement set. Then the state estimation was run again. This procedure was repeated until the state estimator converged and all erroneous measurements were removed. The power plant power flow and voltage measurements were removed from the measurements set based on the power plant breaker status. Table I shows the normalized measurement residuals in the case of the input synchronization error.

After adding the bad data detection, no further problems were encountered in RTDS simulations. The state estimator worked as planned supplying correct state estimates to the coordinated voltage control algorithm.

TABLE I
EXAMPLES OF NORMALIZED MEASUREMENT RESIDUALS DURING BAD DATA DETECTION

Measurement	Erroneous Q_{27}			Input synchronization error		
	z_{real}	z	r^N	z_{real}	z	r^N
P_{23}	–1.35	–1.35	0.00	**8.56**	**–1.35**	**14.29**
P_{27}	11.26	11.26	0.17	11.26	11.26	0
Q_{23}	4.17	4.17	0.00	**2.09**	**4.17**	**10.89**
Q_{27}	**2.85**	**3.85**	**5.54**	2.85	2.85	0
V_6	1.05	1.05	0.00	-	-	-
P_2	0	0	0	0	0	0.00
P_3	6.80	6.80	0.00	6.80	6.80	0.00
P_4	1.15	1.15	0.00	1.15	1.15	0.00
P_5	0.60	0.60	0.00	0.60	0.60	0.00
P_6	–10.00	–10.00	0.00	0	0	0.00
P_7	3.06	3.06	0.07	3.06	3.06	0.00
P_8	4.93	4.93	0.00	4.93	4.93	0.00
P_9	1.94	1.94	0.00	1.94	1.94	0.00
P_{10}	1.12	1.12	0.01	1.12	1.12	0.00
Q_2	0	0	0	0	0	0.00
Q_3	1.89	1.89	0.00	1.89	1.89	0.00
Q_4	0.33	0.33	0.00	0.33	0.33	0.00
Q_5	0.17	0.17	0.00	0.17	0.17	0.00
Q_6	2.00	2.00	0.00	0	0	0.00
Q_7	0.88	0.88	**5.54**	0.88	0.88	0.00
Q_8	1.41	1.41	**5.54**	1.41	1.41	0.00
Q_9	0.55	0.55	**5.54**	0.55	0.55	0.01
Q_{10}	0.32	0.32	**5.54**	0.32	0.32	0.04

C. *Real-life demonstrations*

The operation of the previously presented prototype software was demonstrated in a real Finnish distribution network in May 2010. The demonstration arrangement was somewhat similar to the one shown in Fig. 3. The parts inside the dashed line were replaced by the real distribution network and the prototype software was run on a PC separate to the network management PC running SCADA and DMS. As a safety feature, the operator executed the control commands from the coordinated voltage algorithm manually. As in the Fig. 4, the demonstration network consisted of two medium voltage feeders and one power plant. Instead of the power flow measurements, only current flow measurements were available from the beginning of the feeders. The network and loads were modelled into the MATLAB with the same detail as in the network information system. The load models included hourly load estimates and their standard deviations for each distribution transformer.

During the demonstration of the coordinated voltage control algorithm, some problems were detected in the state estimation. The bad data detection identified the feeder current flow measurements incorrectly as bad data. This was caused by the exceptionally warm weather during the demonstration. The average daily temperature was over 10 °C higher than normally in May. The probability of such weather occurring in May is less than 3 %. High temperature caused a radical drop in heating loads and the bad data detection interpreted low feeder current flows as faulty measurements. This problem could have been avoided if the load temperature dependencies had been taken into account. The state estimator included a load temperature correction feature, but no temperature dependencies were available for the used load models. The bad data detection had to be turned off. Further problems were experienced because of an inaccurate substation voltage measurement. The used voltage measurement had a measurement resolution of 1 % (0.2 kV). This reduced the voltage estimation accuracy significantly. Despite these problems, the coordinated voltage control demonstration was completed successfully [12].

Next, we aimed to verify the results in Fig. 2 by comparing the developed state estimator and the state estimator in a real distribution management system (ABB MicroSCADA Pro DMS 600). Inputs and outputs from the DMS state estimator were saved for later off-line comparison. This required some special arrangements because the DMS 600 does not normally save the state estimates. The DMS 600 source code was edited to save the state estimation results into a database. The state estimation results were then read to the MATLAB through ODBC interface. Finally, the state estimation results and inputs; feeder current flows and substation voltage measurements, were written into a text file. The state estimation was run once an hour and the results were saved for a period of one week. Data was collected from one medium voltage feeder. To find out the true voltages, two voltage measurements were added to distribution transformers at branch ends of the studied feeder. These measurements were done with AMR meters with power quality monitoring functions.

After one week, the data collection PC was retrieved from the distribution network operator's control room and the results were analyzed. We noticed that the DMS state estimator had not corrected the loads to match the feeder current flows. This was caused by human error; the current measurements were not connected to the network model in NIS. Secondly, we discovered that the DMS state estimator had used a different substation voltage measurement than assumed. Thereby, the results of the developed state estimator and DMS state estimator were not comparable.

Even without the above mentioned mistakes, the comparison of state estimators would have been difficult. The demonstration network was lightly loaded during the demonstration and the voltage drops were very small. The differences between estimated and measured node voltages would have been close to the voltage measurement accuracy. The state estimation accuracy could have been verified also by comparing the estimated and measured loads in the distribution transformers. Unfortunately, measurement of the transformer loads was not possible.

IV. Discussion and Conclusions

The branch current based distribution system state estimator was further developed by adding bad data detection. Distribution networks have a very low measurement redundancy, thereby detecting bad data is difficult. In many cases, the only way to identify faulty measurements is to use pseudo-measurements in the bad data detection process. The real-life demonstration of the developed DSSE method proved that the commonly used 3σ bad data detection threshold is inadequate when using load profile data to detect errors in feeder line flow measurements. The bad data detection threshold should be raised and more accurate load models with temperature dependency correction should be used. Further research is needed to find out if these actions are enough to make the bad data detection work. Next, we are going to use AMR measurements to improve the accuracy of load models. After this we can retest the state estimation algorithm.

MATLAB simulations proved that the developed DSSE method is more accurate than the existing Finnish state estimation method. Demonstrating this improvement in a real distribution network is difficult. To see the differences in the voltage estimates, the demonstration network should have big stochastic loads, large voltage drops and very accurate voltage measurements.

Acknowledgements

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